Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Linear Algebra

Subject Code: 5SC01MTC1 **Branch: M.Sc. (Mathematics)**

Semester:1 Date:30/11/2015 Time:10:30 To 1:30 Marks: 70

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Attempt the Following questions. (07)

a) Define: Linearly independent vectors. **(01)**

b) Intersection of two subspaces is subspace but union of two subspaces need not be subspace. Determine whether statement is true or false. **(01)**

c) Let $T: R^2 \to R^3$ define by $T(x,\ y) = (x,\ x+3y,\ 2y)$. Check whether T is linear. **(01)**

d) Are $(2,-1,\frac{3}{2})$ and $(\frac{-1}{7},\frac{1}{14},\frac{-3}{28})$ linearly dependent ? Justify your answer. **(02)**

e) State Cauchy –Schwarz inequality. **(02)**

Q-2 Attempt all questions (14)

a) Define vector space. Show that R_n is vector space. **(06)**

b) Define subspace of vector space. Let V be vector space and $W \subset V$ then show that W is subspace of V if and only if $\alpha u + \beta v \in W$ for all $\alpha, \beta \in R$ and $u, v \in W$. **(06)**

c) What is span of $\{1, x, x^2\}$? **(02)**

OR

Q-2 Attempt all questions (14)

a) For linear transformation $T: R^2 \to R^3$ defined as $T(x,y) = (x, x+y, y)$. Verify rank-nullity theorem. **(06)**

b) For linear transformation $T: R^3 \to R^2$ defined as **(06)**

$T(a,b,c) = (2a + b - c, 3a - 2b + 4c)$ and

$B_1 = \{(1,1,1),(1,1,0),(1,0,0)\}$ & $B_2 = \{(1,3),(1,4)\}$ are basis of R^3 and R^2 respectively, Then find matrix which is associate with linear transformation T.

c) What is orthonormal basis of vector space? **(02)**

Q-3 **Attempt all questions** **(14)**

a) Using gram-Schmidt orthogonalization process find orthogonal basis for M_{22} **(07)**

with $B = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$.

b) If V is finite-dimensional and W is a subspace of V then prove that W is finite dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$. **(07)**

OR

Q-3 **Attempt all questions** **(14)**

a) State and prove Reisz- Reprasentation theorem. **(07)**

b) State and prove rank -nullity theorem. **(07)**

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a) Define linear transformation. **(01)**

b) Prove that if two rows of A are equal then $detA = 0$. **(02)**

c) Define matrix associated with linear transformation. **(02)**

d) Define Jordan block belonging to character root λ. **(02)**

Q-5 **Attempt all questions** **(14)**

a) Define trace of a matrix. **(07)**

For $A, B \in F_n$ and $\lambda \in F$, prove the following

(i) $tr(\lambda A) = \lambda tr(A)$.

(ii) $tr(A + B) = tr(A) + tr(B)$.

(iii) $tr(AB) = tr(A).tr(B)$.

b) Define nilpotent. Let V be a finite dimensional vector space and $T \in A(V)$ be nilpotent with index n_1. Also V_1 be invariant under T. If $u \in V_1$, is such that $T^{n_1}(u) = 0$, $0 < k \leq n_1$, then prove that $T^k(u_0) = u$ for some $u_0 \in V_1$. **(07)**

OR

Q-5 **Attempt all questions** **(14)**

a) Let V and W be finite dimensional vector space over F then prove that the set Hom $(V,W) = \{T: V \to W; T is homomorphism\}$ is also finite dimensional vector space and $\dim Hom(V, W) = \dim V \cdot \dim W$. **(07)**

b) Let V be a finite dimensional vector space over F and $T \in A(V)$ be such that all characteristic roots are in F then prove that there exists a basis of V in which the matrix is triangular. **(07)**

Q-6 **Attempt all questions** **(14)**

a) If V be a finite dimensional vector space. Let $T: V \to V$ be a linear map. Then prove that following are equivalent. **(07)**

(i) T is isomorphism.

(ii) $kerT = \{0\}$

(iii) $Im(T) = V$.

b) Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent. Show that the invariants of T are unique. **(07)**

OR

Q-6 **Attempt all Questions**

a) Let V be a finite dimensional vector space, $T \in A(V)$ and W be a subspace on V invariant under T. Define the linear transformation $\bar{T}$ of T on $\bar{V} = V/_{W}$. Suppose $p(x)$ and $p_1(x)$ are minimal polynomial for T and $\bar{T}$ respectively then $p_1(x)/_{p(x)}$. **(07)**

b) Let V be a finite dimensional vector space over ,$T \in A(V)$ has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F. **(07)**